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AN ANALYTICAL COMPARISON OF SOME ELECTROMAGNETIC SYSTEMS FOR REMOVING MOMENTUM STORED BY A SATELLITE ATTITUDE CONTROL SYSTEM

by Stuart C. Brown

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AN ANALYTICAL COMPARISON OF SOME ELECTROMAGNETIC SYSTEMS FOR REMOVING MOMENTUM STORED BY A SATELLITE ATTITUDE CONTROL SYSTEM

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SUMMARY

The report summarizes the results of a study of methods of removing angular momentum stored in a satellite by producing an electromagnetic torque by coupling the vehicle with the earth's magnetic field. The coupling is achieved by inducing a magnetic moment by means of an electric current in coils on the vehicle. The conventional method of controlling the removal of momentum requires that the local magnetic field be measured with a magnetometer, and that a computation be performed on the measured field components to prescribe the current in each coil. An adaptive method, however, can also be used. In this method, instead of a magnetometer, an accurate determination of changes in stored angular momentum due to selected changes in magnetic moment is used. As a consequence of this approach, the adaptive momentum removal method is applicable only for vehicles in which a relatively high gain attitude control system is required.

The report discusses the properties and limitations of a proportional control system and an on-off control system for each of the two classes of momentum removal systems. A general conclusion can be drawn concerning all the control systems studied. As is to be expected, if the gain of a momentum removal system is too low the performance is sluggish. There is also an upper limit to the choice of gain, however, for with too high a gain, the stored momentum is not removed efficiently. Particular conclusions result from the study of the on-off adaptive system. Whereas this system does not need a magnetometer, it is shown to be more critical than other systems in the accuracy needed for momentum-wheel speed measurements. Other parameters for the adaptive system which were investigated but found to be of less importance include dead zone, sampling interval, and momentum wheel control system time constant.

INTRODUCTION

A number of investigations of the application of electromagnetic torques to control the attitude of satellites have been made (refs. 1 to 6). These torques have been considered both for primary attitude control, as well as for unloading momentum stored by the primary attitude control system. The use of electromagnetic torques is of particular interest for long term missions since no expendable fuel is required. However, the applied magnetic torque can only lie in a plane perpendicular to the earth's magnetic field at the particular position in the orbit. Hence, the attainable accuracy of a primary magnetic control system would be dependent on the disturbing torques present, and would be particularly dependent on those which act in a direction parallel with the earth's magnetic field. The cases of interest for the present report are those for which pointing accuracy requirements are sufficiently high so that a momentum storage device is needed for the primary control system. The magnetic torques are used to reduce the resulting stored angular momentum. This momentum could be stored either by momentum wheels or by components of momentum of unrestrained gyros. For the latter case, the momentum to be counteracted is assumed to be known in component form.

In the previously cited references, similar magnetic unloading systems have been investigated to determine the magnetic moment which would apply a corrective torque as close as possible to a desired direction, and also would lie in the plane perpendicular to the earth's magnetic field. For most cases, this desired direction was taken as that opposite the angular momentum vector which represented the total momentum stored by attitude-control wheels. Systems with proportional magnetic moments involved the calculation of a vector product whose components determined the magnitude of magnetic-moment components to be generated by three orthogonal coils. These references differ chiefly in the simplifications made of the proportional system. The simplifications considered in these reports include several combinations of on-off values of magnetic moment components generated in the vehicle and on-off measured values of inertia wheel momentum and the earth's local magnetic field.

For the present report, vehicles are considered for which magnetic unloading disturbances to the attitude control system and peak power requirements should be minimized. Hence, relatively small torques are applied, and the unloading system acts more or less continuously to counteract the disturbing torques. The performance of systems with two forms of magnetic moment components will be compared. In one system, the components are proportional to a function of the measured magnetic field and stored angular momentum. In the other system, the magnetic moment components depend on the sign of the same continuous function, and hence the on-off system is less of a departure from a proportional system than the systems considered in references 3 and 5. In addition, a closed-form solution of the proportional case will be obtained through use of a simplification of the earth's magnetic field representation. This solution provides qualitative information on system performance.

Whereas the previous studies were concerned with systems which used both measured stored momentum and the earth's magnetic field, the method chiefly investigated in this report requires knowledge only of the former. By perturbing magnetic moment and determining the resulting change in the stored angular momentum, the desired value of total magnetic moment can be obtained. Since a magnetometer is not needed to measure the local earth's magnetic field, but rather the vehicle is made to adjust its momentum without direct knowledge of part of the environment, the method can be called adaptive.

The determination of the magnetic moment is made on the basis of changes in a quadratic scalar function of the stored angular momentum. This function is selected so that the resulting applied magnetic moment is the same as that obtained by the previously discussed magnetometer systems for either the proportional or on-off cases. This similarity is shown to exist as long as certain system imperfections are neglected. In addition, the quadratic function is useful as a Liapunov function to demonstrate stability for the continuous case with either proportional or on-off applied magnetic moments.

The analyses are corroborated by numerical calculations performed with a digital computer. Computations are included which illustrate effects of external disturbance torques. Finally, numerical calculations are made to show the sensitivity of the on-off adaptive system to such quantities as stored momentum measurement errors.

SYMBOLS

A	matrix for transforming a vector from an inertial coordinate system to a rotating coordinate system
$\bar{B}$	earth's magnetic field
B_r	component of the earth's magnetic field assumed to be rotating at a constant rate relative to the vehicle for the simplified analysis
$\bar{B}_1,\bar{B}_2,\bar{B}_3$	numerical values of magnetic field used in the calculations (table I)
C	matrix of elements which represent the vector operations given in equation (12) or (24)
G	diagonal matrix representing first-order dynamics of the moment wheel attitude control system
$\bar{H}$	angular momentum of momentum wheels
H_m	design maximum value of wheel angular momentum
H_n	wheel angular momentum measurement error
$\bar{h}$	normalized wheel angular momentum, $\dfrac{\bar{H}}{H_m}$
$\bar{h}(0)_1$	numerical initial values of $\bar{h}$ used in the calculations (table I)
$\bar{\bar{I}}$	identity dyadic
K	gain term which determines magnetic moment for proportional system

K^*	$\frac{KB^2}{\gamma_{x_3}}$
k	gain term for proportional system with normalized angular momentum (orbital period gauss^2)$^{-1}$
$\overline{M}$	magnetic moment
N	gain term which determines magnetic moment for on-off system
n	gain term for on-off system with normalized angular momentum (orbital period gauss)$^{-1}$
P	orbital period
p	strength of dipole representing the earth's magnetic field
r,θ,φ / r,μ,η	spherical coordinate systems described in the appendix
T	sampling interval
$\overline{T}_a$	applied torque
$\overline{T}_e$	external disturbing torque
t	time
$\overline{t}_e$	normalized disturbing torque, $\frac{\overline{T}_e}{H_m}$
$\overline{t}_{e_1},\overline{t}_{e_2}$	numerical values of normalized disturbing torques used in the calculations (table I)
t_k	time at sampling interval k
V	positive definite reference function, $(1/2)\overline{H} \cdot \overline{H}$
v	positive definite reference function obtained from normalized angular momentum, $(1/2)\overline{h} \cdot \overline{h}$
x,y,z	inertially fixed coordinate system
γ	angular frequency of the periodic portions of the components of $\overline{B}$ and $\overline{t}_e$
$\Delta(\ )$	incremental quantity
$\Delta^2V(t_k)$	$V(t_{k-1}) - 2V(t_k) + V(t_{k+1})$
λ	root of characteristic equation for simplified case

$\lvert\lambda_R\rvert_{min}$	absolute real part of root with minimum absolute real part
σ_t	total rms value of error in calculation of $\Delta^2 V$ due to momentum-wheel measurement noise
σ_n	rms value of measurement error for each momentum wheel
τ_c	momentum-wheel control system time constant, in orbital periods (diagonal elements in $\bar{\bar{G}}$)

Subscripts

I	inertially fixed coordinate system
r	rotating coordinate system
1, 2, 3	rectangular coordinate systems described in the appendix

Superscript

*	dimensionless parameter used for simplified case

Vector Notation

$(\bar{\ })$	vector
$(\bar{\bar{\ }})$	dyadic
$(\bar{\ }) \cdot (\bar{\ })$	scalar or dot product of vectors or dyadics (e.g., ref. 7)
$(\bar{\ }) \times (\bar{\ })$	vector product of vectors
$\mathrm{grad}_{\bar{M}}\, \dot{V}$	gradient of the scalar $\dot{V}$ with respect to $\bar{M}$
$\mathrm{sgn}_{xyz}(\bar{\ })$	vector with unit components determined by the sign of a component of the indicated vector times the corresponding unit base vector in the xyz axis system

ANALYSIS

A magnetic system for unloading stored momentum generates an on-board magnetic moment which couples with the earth's magnetic field to produce a torque on the vehicle in some desired direction. Equations will be given first for a conventional system which measures values of the magnetic field to determine the magnetic moment (magnetometer system). Then the equations will be given for a system which determines the magnetic moment by perturbing components of the moment (adaptive system) and observing the resulting changes in the stored momentum. Both systems are based on the same performance criteria. In addition, equations are given which represent both proportional and on-off magnetic moment control and which are determined from the same control function.

The adaptive system to be described is more dependent on accurate measurements of momentum variations together with associated system parameters than is the magnetometer system. Hence, the effects of several system parameters for the adaptive system will be investigated. These parameters include a first-order representation of the primary attitude control system. A high gain attitude control system has been presupposed so that the frequencies associated with the magnetic unloading system would be expected to be much lower than those for the attitude control system. For the adaptive unloading system, however, the use of a perturbing test signal introduces higher frequency effects so that possible coupling with the primary attitude control system should be considered.

Magnetometer System

A control law determines the relation between the stored angular momentum in the vehicle, the earth's magnetic field, and the applied magnetic moment. This moment determined by the control law will be either proportional to or depend only on the sign of a vector function of these quantities. The equations for the resulting momentum removal will be derived. The stability of the system will then be shown through use of a Liapunov function. In addition, a closed-form solution for the proportional system will be given for a case with simplified assumptions for the variation of the magnetic field.

<u>Determination of magnetic moment</u>.- The magnetic moment from the coils interacts with the earth's magnetic field to produce a torque which tends to rotate the vehicle. The reaction of the attitude control system causes desaturation of the momentum wheels. The coils are assumed to be alined with respect to the vehicle control axes so that a magnetic moment component is produced along each control axis. The actual direction in which the magnetic torque can be applied is restricted by the condition

$$\bar{T}_a = \bar{M} \times \bar{B} \tag{1}$$

that is, the torque can only be applied in the plane perpendicular to $\bar{B}$ regardless of the direction of $\bar{M}$.

A primary high gain attitude control system is assumed to be operating simultaneously to aline the vehicle continuously with some desired direction while the momentum removal system is operating. Even though for a linear attitude control system a constant external torque will cause the vehicle error to increase at a constant angular rate, the control system must be designed so that the rate is very small in order to satisfy pointing accuracy requirements (ref. 8). Since this resulting attitude error is proportional to the momentum wheel speed, a signal based on momentum wheel speed only is sufficient for the unloading system.

A desired direction of the applied magnetic torque would be opposite to that of the wheel angular momentum. Since the applied torque must lie in the plane perpendicular to $\bar{B}$, the desired torque is selected to be in the opposite direction from the projection of $\bar{H}$ in the plane perpendicular to $\bar{B}$

$$\bar{T}_a = -K_1 \bar{H}_{Proj}$$

$$= -K_1 \left[\bar{H} - \frac{(\bar{H} \cdot \bar{B})\bar{B}}{B^2} \right] \tag{2}$$

or

$$\bar{T}_a = \frac{K_1}{B^2} \left[(\bar{H} \times \bar{B}) \times \bar{B} \right] \tag{3}$$

Moreover, it is desirable to constrain $\bar{M}$ to the plane perpendicular to $\bar{B}$ since a component of $\bar{M}$ parallel to $\bar{B}$ does not contribute toward the production of torque. A comparison of equations (3) and (1) indicates the conditions are satisfied if

$$\bar{M} = \frac{K_1}{B^2} (\bar{H} \times \bar{B}) \tag{4}$$

Essentially this form of control law has been used in a number of previous investigations (refs. 1 to 5). A simplification which has been made in several of the cited references is to neglect the variation of the $1/B^2$ term in equation (4). The magnitude of $\bar{B}$ can vary at most by a factor of 2 for a circular orbit (polar orbit). While this maximum variation could be expected to have some effect on performance, the additional complication of the $1/B^2$ term is usually not needed. Thus, equation (4) simplifies to

$$\bar{M} = K(\bar{H} \times \bar{B}) \tag{5}$$

Each component of $\bar{M}$ in equation (5) can be expressed as

$$M_i = K(H_j B_k - H_k B_j) \tag{6}$$

where the i, j, and k components are taken in cyclic order. These equations are continuously computed in the vehicle to obtain the proper currents for each magnetic coil. The resulting applied torque is

$$\bar{T}_a = K(\bar{H} \times \bar{B}) \times \bar{B} \tag{7}$$

The previous equations have been for cases with three orthogonally mounted coils. If only two magnetic coils were used with the same control law, the magnetic moment would be constrained to lie in a plane which would not necessarily contain the desired direction of $\bar{M}$. A knowledge of the reduced performance of a two-coil system is useful from the reliability standpoint, however. If one coil of the three-coil system failed, the remaining system would still operate satisfactorily provided the wheels and coils had sufficiently large capacity.

The implementation of the three-coil system can be simplified if an on-off current is applied to the coils in response to the sign of a signal rather than a current which is proportional to the signal. For this on-off case, equation (5) is modified to be

$$\bar{M} = N \, \mathrm{sgn}_{xyz}(\bar{H} \times \bar{B}) \tag{8}$$

where $\mathrm{sgn}_{xyz}(\bar{\ })$ is a vector with unit components whose directions are determined by the sign of the components of $(\bar{\ })$ along the xyz axes. Note that the direction of $\bar{M}$ will be affected by the orientation of the xyz axes. For the on-off case in component form, equation (8) becomes

$$M_i = N \, \mathrm{sgn}(H_j B_k - H_k B_j) \tag{9}$$

The resulting applied torque is

$$\bar{T}_a = N[\mathrm{sgn}_{xyz}(\bar{H} \times \bar{B})] \times \bar{B} \tag{10}$$

The computation could be simplified further by using only the signs of the $\bar{H}$ and $\bar{B}$ components. However, this simplification would be expected to be useful when the magnetic unloading is needed only occasionally. When selected dead zones of these components are exceeded, the direction of the resulting applied moment can be the reverse of that given by equation (9).

Equations of motion.- Changes in control wheel angular momentum can be directly related to torque acting on the vehicle for an inertially oriented vehicle with a high gain attitude control system.

$$\dot{\bar{H}} = \bar{T}_a + \bar{T}_e$$

After the substitution of equation (1)

$$\dot{\bar{H}} = \bar{M} \times \bar{B} + \bar{T}_e \tag{11}$$

The substitution of equation (5) or (8) into equation (11) yields

$$\dot{\bar{H}} = K(\bar{H} \times \bar{B}) \times \bar{B} + \bar{T}_e \tag{12}$$

for the proportional system or

$$\dot{\bar{H}} = N \left[\mathrm{sgn}_{xyz}(\bar{H} \times \bar{B}) \right] \times \bar{B} + \bar{T}_e \tag{13}$$

for the on-off system.

Equation (12) can be arranged into the form

$$\dot{\overline{H}} = K\overline{\overline{C}} \cdot \overline{H} + \overline{T}_e \tag{14}$$

where the elements of $\overline{\overline{C}}$ are

$$C = \begin{bmatrix} -(B_y^2 + B_z^2) & B_xB_y & B_xB_z \\ B_xB_y & -(B_x^2 + B_z^2) & B_yB_z \\ B_xB_z & B_yB_z & -(B_x^2 + B_y^2) \end{bmatrix}$$

A block diagram of the on-off magnetometer system is shown in figure 1(a).

Stability of the system.- System stability can be evaluated through the selection of a suitable positive definite (Liapunov) function (ref. 9). A natural selection is one which is proportional to the square of the angular momentum of the inertia wheels,

$$V = (1/2)\overline{H} \cdot \overline{H} \tag{15}$$

Differentiating (stability is determined from an examination of $\dot{V}$) with respect to time gives

$$\dot{V} = \dot{\overline{H}} \cdot \overline{H} \tag{16}$$

For the proportional case, the substitution of equation (12) with $\overline{T}_e = 0$ yields

$$\dot{V} = K[(\overline{H} \times \overline{B}) \times \overline{B}] \cdot \overline{H} \tag{17}$$

Equation (17) can be rearranged so that

$$\dot{V} = -K(\overline{B} \times \overline{H}) \cdot (\overline{B} \times \overline{H}) \tag{18}$$

Equation (18) is seen to be negative semidefinite. Hence, the possibility of equilibrium points for other than $\overline{H} = 0$ must be investigated. When $\dot{V} = 0$ for values of $\overline{H}$ unequal to zero $(\overline{B} \times \overline{H} = 0)$, then $\dot{\overline{H}} = 0$, but $\dot{\overline{B}} \neq 0$ since the magnetic field, $\overline{B}$, is a time varying quantity. Hence, $\overline{B}$ cannot remain perpendicular to $\overline{H}$, and therefore $\dot{V}$ cannot remain equal to zero for values of $\overline{H}$ other than $\overline{H} = 0$. Thus, the origin is the only equilibrium point and it is asymptotically stable.

For the on-off system, the substitution of equation (13) with $\overline{T}_e = 0$ gives

$$\dot{V} = N[\mathrm{sgn}_{xyz}(\overline{H} \times \overline{B}) \times \overline{B}] \cdot \overline{H} \tag{19}$$

Equation (19) can be rearranged to give

$$\dot{V} = -N[\mathrm{sgn}_{xyz}(\bar{B} \times \bar{H})] \cdot (\bar{B} \times \bar{H}) \tag{20}$$

Asymptotic stability for the on-off system is also assured since, although $\dot{V}$ is again negative semidefinite, $\bar{H} = 0$ is the only possible equilibrium point as long as $\bar{B}$ is time varying.

Unfortunately, no information concerning the degree of stability is obtained for either the proportional or on-off system since $\dot{V} = 0$ at other points in the phase space besides the origin. Furthermore, maximum variations due to disturbing torques cannot be estimated through use of this V function.

Simplified analysis.- A closed-form solution for the proportional control can be obtained when a simplification is made of the representation of the earth's magnetic field. This part of the analysis will be useful principally for obtaining trends concerning the effect of gain on degree of stability. The earth's magnetic pole is assumed to be at the geographic pole, the magnetic field is represented by that obtained from a simple dipole, and the vehicle is assumed to be in a circular orbit. With these assumptions, the component of the earth's magnetic field in one direction is constant with respect to the inertially fixed vehicle. As shown in the appendix, the other two components of the earth's magnetic field can be approximately represented by a constant magnitude vector rotating at a rate equal to twice the orbital rate. With these assumptions, a set of axes rotating at a constant rate can be selected in which $\bar{B}$ is constant. Hence, the analysis is reduced to that of a constant coefficient linear system with this reference frame.

The transformation of a vector from the fixed set of axes to the rotating set can be expressed as

$$(\bar{\ })_r = \bar{\bar{A}}(t) \cdot (\bar{\ })_I \tag{21}$$

Dyadic terms in the rotating system can be expressed by means of the transformation dyadic from the inertially fixed term as

$$(\bar{\bar{\ }})_r = \bar{\bar{A}}(t) \cdot (\bar{\bar{\ }})_I \cdot \bar{\bar{A}}^T(t) \tag{22}$$

Thus, in the rotating system, the earth's magnetic field vector transforms to

$$\bar{B}_r = \bar{\bar{A}}(t) \cdot \bar{B}(t)_I \tag{23}$$

With the distrubing torque equal to zero, equation (12) can be expressed in the rotating coordinate system as

$$\left.\frac{d\bar{H}_r}{dt}\right)_r + \bar{\gamma} \times \bar{H}_r = K(\bar{H}_r \times \bar{B}_r) \times \bar{B}_r \tag{24}$$

Equation (24) can be solved in closed form. The equation is first rearranged to the form

$$\left.\frac{d\bar{H}_r}{dt}\right)_r = K\bar{\bar{C}}_r \cdot \bar{H}_r \tag{25}$$

For convenience, an x_3, y_3, z_3 rotating coordinate system is selected so that the inertially fixed component of B lies along the x axis and the rotating component of the magnetic field, B_r, lies along the z axis. For these orientations, the elements of C_r have the following form:

$$C_r = \begin{bmatrix} -B_{z_3}^2 & 0 & B_{x_3}B_{z_3} \\ 0 & -(B_{x_3}^2 + B_{z_3}^2) & \gamma_{x_3}/K \\ B_{x_3}B_{z_3} & -\gamma_{x_3}/K & -B_{x_3}^2 \end{bmatrix} \tag{26}$$

The elements of $\bar{C}_r$ form a 3×3 matrix with constant coefficients since the components of $\bar{B}$ are constant in the rotating reference frame. Hence, equation (25) can be solved in closed form, and the following third degree characteristic equation is obtained:

$$\lambda^3 + 2KB^2\lambda^2 + (K^2B^4 + \gamma_{x_3}^2)\lambda + KB_r{}^2\gamma_{x_3}^2 = 0$$

A convenient dimensionless form for this equation is

$$\lambda*^3 + 2K*\lambda*^2 + (K*^2 + 1)\lambda* + K*\frac{B_r{}^2}{B^2} = 0 \tag{27}$$

where

$$\lambda* = \lambda/\gamma_{x_3}$$

$$K* = \frac{KB^2}{\gamma_{x_3}}$$

For the on-off case, a similar simplification cannot be made since the orientation of the inertial xyz system must still be considered (sgn_{xyz} term in eq. (13)).

The solution for a particular orientation of a two-coil system can also be obtained. This orientation is such that magnetic moments are applied in the yz plane only. Three-axis magnetometer information is still used. After expressing equation (24) in component form with the M_x component set equal to zero, the following third-order characteristic equation is obtained for the two-coil system:

$$\lambda^3 + K(2B^2 - B_r{}^2)\lambda^2 + (K^2B^4 - K^2B^2B_r{}^2 + \gamma_{x_3}^2)\lambda + K\gamma_{x_3}^2B_r{}^2 = 0 \tag{28}$$

In dimensionless form, equation (28) becomes:

$$\lambda^{*3} + \left(2 - \frac{B_r^2}{B^2}\right) K^*\lambda^{*2} + \left[K^{*2}\left(1 - \frac{B_r^2}{B^2}\right) + 1\right] \lambda^* + K^* \frac{B_r^2}{B^2} = 0 \qquad (29)$$

Adaptive System

A second method of control involves an indirect evaluation of equation (5) or (8); that is, a measurement of the earth's magnetic field is not needed. In this approach, the magnetic moment is perturbed, and from the resulting variations in a selected scalar reference function of $\overline{H}$, the value of $\overline{M}$ is obtained which is the same as that which would be calculated through use of the previous magnetometer system. The resulting change in angular momentum is the same as that for the magnetometer system except for the effects of the magnetic moment test signal perturbations. Equations to show the effects of other system parameters will also be given. While the most useful form of this system is felt to be a sampled on-off form, a continuous form will be shown first in order to indicate that the previous control law is being implemented.

Continuous adaptive system.- To obtain a system with performance equivalent to that of the magnetometer system, a means for evaluating the quantity $\overline{B} \times \overline{H}$ without a direct measurement of $\overline{B}$ must be found. This can be done by first writing the previously used time derivative of a scalar function of momentum in terms of $\overline{M}$ through use of equations (16) and (11).

$$\dot{V} = (\overline{B} \times \overline{H}) \cdot \overline{M} + \overline{T}_e \cdot \overline{H} \qquad (30)$$

If the partial derivative of $\dot{V}$ is taken with respect to $\overline{M}$,

$$\text{grad}_{\overline{M}}\, \dot{V} = \overline{B} \times \overline{H} \qquad (31)$$

For a proportional system, the desired value of M can be obtained in terms of the partial derivative from a comparison of equations (31) and (5). The resulting equation is

$$\overline{M} = -K\, \text{grad}_{\overline{M}}\, \dot{V} \qquad (32)$$

Thus varying one component of $\overline{M}$ at a time, while computing the resulting change in $\dot{V}$ (a function of $\dot{\overline{H}}$ and $\overline{H}$) gives the desired $\overline{M}$ according to equation (32).

A comparison of equations (31) and (8) shows that the desired value of $\overline{M}$ in terms of the partial derivative for the system with on-off magnetic components appears as:

$$\overline{M} = -N\, \text{sgn}_{xyz}(\text{grad}_{\overline{M}}\, \dot{V}) \qquad (33)$$

The resulting motion is obtained by the substitution of equation (32) or (33) into equation (11).

Sampled adaptive system.- The adaptive system is more readily implemented in a discrete form. A block diagram of the discrete adaptive system for the on-off case is shown in figure 1(b). Starting with a given set of three magnetic coil currents, two sampling intervals are required to provide a discrete test signal for the effect of one magnetic coil. For the first sampled interval, all components are at a previously determined level. For the second interval, the current for the coil being tested is varied by a preselected amount. A comparison of the change in V over the sampling interval after the variation in coil current with that over the sampling interval before the variation determines the new value for the component of magnetic moment. The discrete equivalent of equation (32) for one component of $\bar{M}$ for the proportional case is

$$M_i = \frac{-K\Delta\left(\frac{\Delta V}{\Delta t}\right)}{\Delta M_i}$$

or, in terms of sampling instants,

$$M_i(t_{k+1}) = \frac{-K}{T} \frac{[V(t_{k-1}) - 2V(t_k) + V(t_{k+1})]}{M_i(t_k) - M_i(t_{k-1})} \tag{34}$$

The time, t_k, is the sampling instant at which the coil current is varied a preselected amount. This computed component of magnetic moment is then applied for the next five sampling intervals. The other two components are similarly tested in sequence with two sampling intervals required for each component. Thus a total of six sampling intervals is needed for each cycle. The cycle is then repeated. An alternate form for equation (34) which is equivalent for small changes in $\bar{H}$, and somewhat easier to implement, is

$$M_i(t_{k+1}) = \frac{-K}{T} \frac{\sum_j H_j(t_k)[H_j(t_{k-1}) - 2H_j(t_k) + H_j(t_{k+1})]}{M_i(t_k) - M_i(t_{k-1})} \tag{35}$$

For the on-off case, the same switching function is used, and only the proper sign of the magnetic moment needs to be determined. The equation evaluated for the on-off case is a modification of equation (34)

$$M_i(t_{k+1}) = -N \operatorname{sgn} \frac{V(t_{k-1}) - 2V(t_k) + V(t_{k+1})}{M_i(t_k) - M_i(t_{k-1})} \tag{36}$$

Since sufficiently accurate measurements of changes in angular momentum during the sampling intervals will be a problem, coil current changes should be as large as possible but should not disturb the tracking ability of the momentum-wheel control loop. These conflicting requirements suggest that a good compromise would be to reduce the current of the coil being tested to zero for the second sampling interval to obtain the largest variation in wheel speed possible. With this selection, equation (36) becomes

$$M_i(t_{k+1}) = N \operatorname{sgn} \frac{[V(t_{k-1}) - 2V(t_k) + V(t_{k+1})]}{TM_i(t_{k-1})} \qquad (37)$$

or

$$M_i(t_{k+1}) = N \operatorname{sgn} \left[\frac{\Delta^2 V(t_k)}{TM_i(t_{k-1})}\right] \qquad (38)$$

In order to avoid continuously supplying power to the coils when the angular momentum is reduced, a dead zone for the $\Delta^2 V$ calculation can be added. However, the magnetic moment would still need to be applied for one sampling interval of the adaption cycle to provide a test signal. For subsequent calculations, the dead zone will be expressed as $|\Delta^2 V/NT|_{dz}$. The limiting case of a dead zone for very small sampling interval is designated as $|\Delta\dot{V}/N|_{dz}$ and is given by the following relation:

$$\lim_{T\to 0} \left|\frac{\Delta\left(\frac{\Delta V}{T}\right)}{N}\right|_{dz} = \left|\frac{\Delta\dot{V}}{N}\right|_{dz}$$

The Liapunov function used to show stability for the continuous case can not be used for indicating stability in the sampled case. Consider the on-off system. For a sampled magnetic moment, the equivalent of equation (20) is

$$V(t_{k+1}) - V(t_k) = -N\int_{t_k}^{t_{k+1}} \{\operatorname{sgn}_{xyz}[\bar{B}(t_k) \times \bar{H}(t_k)]\} \cdot [\bar{B}(t) \times \bar{H}(t)]dt \qquad (39)$$

If average values of $\bar{B}(t)$ and $\bar{H}(t)$ are taken over the sampling interval, equation (39) becomes

$$V(t_{k+1}) - V(t_k) \approx -NT\{\operatorname{sgn}_{xyz}[B(t_k) \times H(t_k)]\} \cdot [B(t_{k+1/2}) \times H(t_{k+1/2})] \qquad (40)$$

Since the terms in the first product are not evaluated at the same instant of time as the second product, the result is not necessarily positive or zero; hence, $V(t_{k+1}) - V(t_k)$ is not negative semidefinite. An additional delay effect due to sampling is also introduced since the value of one component of magnetic moment is held while the other two components are being tested.

The effect of wheel-speed measurement errors on system performance will also be investigated. Since the calculation of $\Delta^2 V$ requires the measurement of incremental changes in wheel speed, effects of wheel-speed inaccuracies are more important for the adaptive than for the magnetometer system. The measured value of one component of momentum consists of the actual value H_i plus a small measurement error H_{n_i}. If the second-order term of H_{n_i} is neglected, the squaring operation for the determination of V with measurement noise present is approximated by

$$(1/2)(H_i + H_{n_i})^2 \approx (1/2)H_i^2 + H_iH_{n_i} \tag{41}$$

The measurement error, H_{n_i}, for each momentum wheel is assumed to have a gaussian probability distribution with a given rms value, σ_n. The measurement errors at successive sampling instants and the errors for each momentum wheel are assumed to be uncorrelated with each other. The portion of total rms error which results from the contribution to Δ^2V from one momentum wheel for the calculation indicated in equation (37) is obtained by means of the approximation shown in equation (41). In addition, the changes in each H_i for the three sampling intervals in which Δ^2V is computed are assumed to be small in comparison with the total values. Only an average value of H_i (at the middle sampling interval) will be used to obtain the effect of measurement error in the product $H_iH_{n_i}$. The resulting equation for the effect of measurement error for one momentum wheel is

$$\sigma_{t_i} = H_i\sigma_n\sqrt{1 + 2^2 + 1}$$

$$= H_i\sigma_n\sqrt{6}$$

The total rms error in the calculation of Δ^2V which results from the three momentum wheels is

$$\sigma_t = \sigma_n[6(H_x^2 + H_y^2 + H_z^2)]^{1/2}$$

or

$$\sigma_t = 2\sigma_n\sqrt{3V} \tag{42}$$

Hence, in the determination of M_i (eq. (38)), the momentum wheel measurement errors, each of whose rms value is σ_n, result in an rms error of σ_t due to the calculation of Δ^2V.

Control system dynamics.- An additional effect to be considered for the adaptive system is the momentum wheel control system dynamics, as the adaptive system is very dependent on momentum wheel speed changes. Since a vehicle high gain control system is assumed, the wheel control system dynamics can be represented by a simple time constant, and interaxis coupling will be neglected. Equation (11) is modified to

$$\bar{\bar{G}} \cdot \ddot{\bar{H}} + \dot{\bar{H}} = \bar{M} \times \bar{B} + \bar{T}_e \tag{43}$$

where $\bar{\bar{G}}$ contains only diagonal elements which represent the control system time constants for each axis.

For the case with magnetic moment sampled, the resulting angular momentum at each sampling interval can be obtained by the stepwise solution of equation (43) between sampling intervals. A component of $\bar{M}$ and resulting components of $\ddot{\bar{H}}$ change discontinuously at each sampling interval.

Normalized Form of the Parameters

For the calculated results, it will be convenient to use a normalized form of the wheel angular momentum such that the maximum allowable momentum is normalized to unity. The normalized angular momentum is designated by

$$\bar{h} = \frac{\bar{H}}{H_m}$$

where H_m is the maximum design angular momentum of one of the wheels.

In addition, it will be convenient to normalize time to an orbital period since the assumed disturbances and $\bar{B}$ variations in the subsequent calculations are periodic with respect to this time interval. The corresponding form of the disturbing torque is

$$\bar{t}_e = \frac{\bar{T}_e}{H_m}$$

The dimension of the normalized torque is $(\text{time})^{-1}$; for this case, the unit of $\bar{t}_e$ is $(\text{orbital periods})^{-1}$.

The gain terms, K (eq. (12) for the proportional system) and N (eq. (13) for the on-off system), contain the conversion units for magnetic moment. For the normalized form, K remains the same while N is modified by the maximum angular momentum

$$k = K$$

$$n = \frac{N}{H_m}$$

It is convenient to list these gains in terms of the earth's magnetic field units and the time unit selected. Thus, for the subsequent calculations, the units of k are $(\text{orbital period gauss}^2)^{-1}$ and the units of n are $(\text{orbital period gauss})^{-1}$.

RESULTS AND DISCUSSION

Calculated variations in stored momentum for the magnetometer and adaptive magnetic momentum-unloading systems described in the analysis section will be shown. The calculations were made with a digital computer by numerical integration of the appropriate equations of motion. Results for both proportional and on-off applied magnetic moments will be compared. As was previously noted, the on-off case is more of interest for the adaptive system since the on-off case will tolerate less stringent wheel-speed accuracy requirements. The magnetometer and adaptive systems are equivalent so long as sampling, magnetic-moment test signals, and system imperfections are neglected for the adaptive system. The results shown initially (figs. 2 to 8) apply both to the magnetometer and adaptive systems since the applied magnetic moment is the same for both systems as described in the magnetometer and continuous adaptive portions of the analysis section. The normalized form of equation (12) was used for calculating the time histories of angular momentum for the proportional system, and equation (13) for the on-off system. The calculations in figure 4 pertain to the simplified representation of the magnetic field described in the analysis section and were obtained from equations (27) and (29). The remaining results (figs. 9 to 12) indicate effects of wheel angular momentum measurement errors, sampling intervals, and attitude control system time constants for the adaptive system. Effects of these parameters on the applied magnetic moment were described in the sampled adaptive system and control system dynamics portions of the analysis section. The resulting variations in $\bar{h}$ were calculated by the substitution of the appropriate magnetic moment into the normalized form of equation (43).

Only variations in wheel speed angular momentum are considered since a high gain attitude control system is assumed which keeps the vehicle continuously pointed in the desired inertial direction so that vehicle angular momentum is small. During the relatively short time for attitude control system transients, the unloading system would not necessarily apply torque in the desired direction. However, the unloading system could be turned off while attitude error and/or error rate exceed a certain value. Even if the unloading system were left on, since the magnitude of the unloading torques is small in comparison with the momentum wheel torques, the effect on the attitude control system motions would be small.

For an inertially oriented vehicle, the predominant variation in the earth's magnetic field relative to the vehicle is at a frequency of twice the orbital frequency for a circular orbit (refs. 1 and 10). This variation is also typical for the principal external disturbing torques (gravity) which act on the vehicle. Thus, the form to be used in the calculations for the magnetic field and disturbing torques is a constant term plus a sinusoidal term whose frequency is twice the orbital frequency. More complete descriptions of expected variations in the magnetic field are given in references 1 and 10. The sets of numerical values of $\bar{B}$ and $\bar{t}_e$ used in the subsequent results are given in table I and indicated with the appropriate figures. The variations in the earth's magnetic field parameters represent a range from that assumed by the simplified analysis of the previous section to that more representative of the earth's magnetic field for a 35° inclined orbit for an inertially fixed vehicle. The two sets of external torque values represent one case in which the torque is alined with the magnetic field and another in which the orientation is arbitrary with respect to the field.

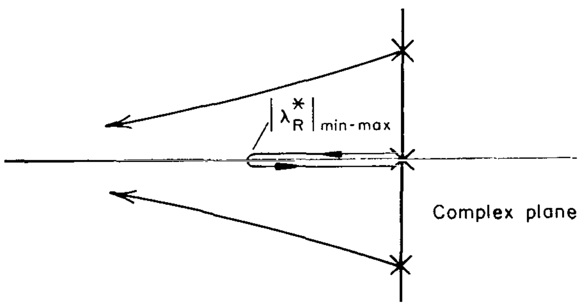
Transient Responses of Proportional and On-Off Systems

Typical transient responses, which result from initial pointing errors of wheel angular momentum for the proportional and on-off systems with and without disturbing torques, are given in figures 2 and 3. Shown are the wheel angular momentum for each control axis and the scalar reference function of the angular momenta. The initial angular momentum and disturbing torques selected are out of phase with the magnetic field vector. The gains of each system have been adjusted to give a similar settling time for the magnitude of the initial angular momentum used. On the average, the linear system produces an exponential-type decay, whereas the on-off system produces a linear-type decay. The time variation of the earth's magnetic field results in appreciable oscillations about this average motion.

Effect of Magnetic Moment Gain on Transient Response

The effect of magnetic moment gain will first be shown for the simplified case described in the analysis section for which trends can most clearly be seen. The effect of gain with a more realistic variation of $\bar{B}$ and with on-off magnetic moments used will then be indicated.

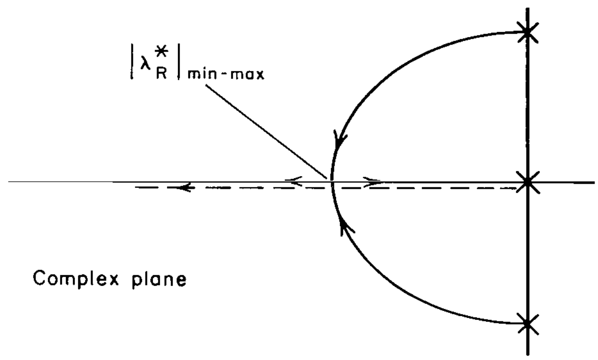
A closed-form solution can be obtained for the proportional case in which the $\bar{B}$ vector is constant in magnitude and rotates at a constant angular rate relative to the vehicle (see Analysis). An indication of the transient response can be obtained by an examination of the characteristic roots (eq. (27) for the three-coil system or (29) for a particular orientation of a two-coil system). Typical variations of the roots for either system with the magnetic moment gain are shown in sketch (a) for the system for small and moderate values of B_r/B. The direction of increasing gain is shown by the arrows. The curve is not a conventional root locus form since the gain term cannot be factored from the rest of the expression in equation (27). As indicated on the sketch, the real root increases to a maximum absolute value and then decreases toward zero for very high gain. When this maximum occurs, the real parts of the complex roots are always more negative than the real root. Thus, a gain can be selected for which the root with the minimum absolute real part has a maximum absolute value. This behavior is a consequence of the restriction on the direction of application of the applied torque. If the gain is too low, then initial errors are not corrected very fast as would be expected. If the gain is increased, the response for initial angular momentum in directions approximately perpendicular to the magnetic field is faster. Hence, the angular momentum vector tends to orient more rapidly toward the rotating $\bar{B}$ and becomes closer to it. If the gain is too high, the angular momentum vector becomes alined very closely to the $\bar{B}$ and this remaining portion of the angular momentum is removed very slowly. This behavior is also indicated by the fact that the eigen-vector associated with the minimum root is approximately in the direction of $\bar{B}$ and it becomes more parallel with $\bar{B}$ as the gain is increased. Thus even for this case with symmetry of the control system parameters about all axes, the three characteristic roots are not all equal because of the restriction on the direction of the applied torque. Hence, caution



Sketch (a)

should be used in including magnetic torque effects in the analysis of a single-axis control system since an "average" root applicable to a particular inertially fixed direction is difficult to define.

For both two- and three-coil systems in which almost the entire $\overline{B}$ vector rotates with respect to the vehicle, $(B_r/B) \rightarrow 1$, all roots become real when the gain becomes sufficiently high, as shown in sketch (b). For the gain at which the two complex roots become real, the single real root is always more negative so that $|\lambda_R^*|_{min-max}$ always occurs at the point indicated on the sketch.



Sketch (b)

The variation of the minimum roots with B_r/B for the two- and three-coil cases is shown in figure 4. The corresponding gains are also shown. A given value of gain for the two-coil system represents only two-thirds of the magnetic moment that would be applied by the three-coil system. The discontinuities in the slope of the curves near $B_r/B = 0.9$ occur because of the change in form of the roots at $|\lambda_R^*|_{min-max}$ shown in the previous two sketches. The two-coil system results in a larger value of the parameter $|\lambda_R^*|_{min-max}$ for portions of the B_r/B range, although considerably higher gain levels are required. However, the control law selected does not necessarily optimize the minimum root parameter shown in figure 4. The minimum root parameter could be improved by a modification of the control laws if the knowledge about the magnetic field variation is improved. First the direction of rotation of the $\overline{B}$ vector would be measured. Then a weighting factor would be used to give the component of momentum perpendicular to the $\overline{B}$ rotation vector more emphasis than the component of momentum parallel with the $\overline{B}$ rotation. The system would then operate as before with this weighted wheel-momentum vector in place of the actual momentum vector. However, the increase in performance would not be worth the added complexity, in general.

The effect of gain variation on the characteristic roots for the simplified case has previously been examined. Some typical time histories will now be shown so that the effect of gain for more realistic cases can be determined. The initial angular momentum and disturbing torque selected will be alined with the direction of $\overline{B}$, since these conditions represent the most difficult situation to control. An example transient to show the effect of gain is given in figure 5. The effect of gain for a constant magnitude of $\overline{B}$ (the assumption made for the simplified analysis) is shown in figures 5(a) and 5(c). The lower gain case (fig. 5(a)) is that which corresponds to the minimum root shown in the previous figure and results in the better transient response. The effect of varying the magnitude of $\overline{B}$ by ±20 percent is shown in figures 5(b) and 5(d). The same effect of gain on transient response also occurs.

The performance of the on-off system for two magnetic moment levels and for the same values of $\bar{B}$ and initial angular momentum used for the previous proportional case are shown in figure 6. The two levels were selected so that about the same average magnetic moment would be obtained during the transient as was obtained for the previous proportional case. The same trend occurs, in that performance deteriorates for the larger gain.

Effects of a disturbing torque in the direction of $\bar{B}$ are shown in figures 7 and 8 for the proportional and on-off cases, respectively. In the presence of a periodic disturbing torque, the motions reach a steady-state oscillation, whose period is the same as that of the disturbing torque and magnetic field. Since the direction of the torque selected is parallel with that of the earth's magnetic field, the mode which corresponds to the minimum root previously discussed is the one predominately excited. Again, poorer performance is obtained with the larger gain cases.

It should be mentioned again that the previous examples in figures 5 to 8 are special cases of initial conditions and disturbing torques alined with $\bar{B}$, for which the resulting trends with gain are different from those usually expected. For other initial conditions and disturbing torques, the higher gains will tend to give better performance. However, consideration should also be given to these cases for which the performance is not so good. One consequence of this limitation on transient response is that a larger size of wheels for momentum storage would be needed if the predominant disturbing torques tend to be in the direction of the magnetic field vector.

The fact that the trends shown in figures 6 through 8 are the same as those illustrated in figure 5 lends confidence to the judgment that the gains for the real situation should be selected on the same basis as those for the simplified case; that is, by selecting gains which give the fastest response from initial conditions and disturbing torques with typical magnitudes and with directions parallel with the magnetic field, the best "worst case" performance is obtained. This determination is similar to the determination of the $|\lambda_R^*|_{min\text{-}max}$ root for the simplified case.

The problem with the high gain can also be reduced by the selection of a suitably sized dead zone for the control function. The dead zone is particularly needed for the on-off control since the "effective gain" of the on-off control becomes large for small values of the control function.

Effect of Control System Parameters On Adaptive System

The previous results have been valid for both the magnetometer and adaptive systems, since their equivalence was shown in the analysis section for cases in which no system imperfections were considered. Effects of imperfections of the adaptive system will now be investigated. The attitude control system is still assumed to maintain the vehicle pointing accuracy within some selected requirement. Only the on-off adaptive system will be considered since, in addition to being simpler than the proportional system, it utilizes a larger test signal and, hence, has better characteristics in the presence of wheel-speed measurement errors. The control system parameters to be investigated are control function dead zone, sampling interval, momentum wheel-speed measurement errors, and attitude control system time constant.

Dead zone.- A dead zone of the control function (described in the analysis section) is useful particularly for an on-off system in order to conserve coil current when the wheel angular momentum is small, or when it cannot be reduced very effectively because of the direction of the magnetic field. Typical effects of dead zone on transient response with the initial angular momentum not alined with the magnetic field are shown in figure 9. The effect of adding an external disturbance torque is given in figure 9(b). The curves shown are for the limiting case of very small sampling intervals. Because of the test signal required, the dead zone only reduces effectively the applied magnetic moment component to one-sixth of the "on" value, rather than zero. For the directions of initial conditions and external torques considered, increasing the dead zone causes a deterioration in speed of angular momentum reduction. However, for a case such as that previously investigated in which the initial momentum vector is parallel with the magnetic field and a relatively large gain is used, a dead zone would be beneficial for the transient response in order to effectively reduce the gain. In addition, the total coil current consumed can be considerably reduced with only a small penalty in momentum removal rate. For the dead zone increment $|\Delta\dot{V}/n|_{dz} = 0.08$, the total coil current for the time interval shown in figure 9(a) is reduced to 0.33 of the amount used with no dead zone present and no external torque. For dead zone $|\Delta\dot{V}/n|_{dz} = 0.32$, the total current is further reduced to 0.20 of the no dead-zone case.

Sampling interval.- The effect of sampling interval variations without dead zone present on transient response is shown in figure 10. The effect of external torque is added in figure 10(b). Note that variations in the sampling interval include variations in the test signal duration also. As would be expected, an increase in sampling interval causes a deterioration in transient performance.

Momentum wheel-speed measurement errors.- Typical transients with wheel-speed measurement errors present are shown in figure 11. The description of the manner in which the measurement error is included is given in the analysis section. Note that repeat runs of these time histories would vary because of the variations in measurement error for each run. Although sufficient runs were not computed to obtain true averages, a sufficient number were made so that "typical" time histories could be presented. For each average value of error, the sampling interval and the dead zone are adjusted to minimize simultaneously the deteriorating effects of measurement error, sampling interval, and dead zone. The rms value of 1 percent represents about an upper limit for satisfactory performance.

Control-system time constant.- For the previous calculations, coupling between the magnetic unloading system and the primary attitude control system was neglected altogether. To explore possible coupling effects for the adaptive case, a first-order representation of the attitude control system dynamics was used as explained in the analysis section. The effect of this parameter, with and without measurement error present, is shown in figure 12. The same time constant has been assumed for each axis and no interaxis coupling terms have been included. Hence, the quantity G given in the analysis section (eq. (43)) contains only diagonal elements equal to τ_c. In contrast to the previous figure, the sampling interval and dead zone were held constant for this comparison. As indicated by the figure, the deteriorating effect of the time constant is greater when measurement error is present. Even with the measurement error present, the effect of the time constant is seen to be small so long as it does not exceed about 0.005 orbital period, which is conservative for a high gain system.

CONCLUDING REMARKS

An investigation has been made of the use of magnetic torques for desaturating stored angular momentum in an attitude control system of an inertially oriented satellite. An important aspect of the problem is that the direction of the control torque is restricted at any given instant of time. Thus, the time varying properties of the magnetic field relative to the vehicle must be used, although only instantaneous information on the magnetic field is available for the control function which determines the magnetic moment.

A consequence of the restriction on the applied torque direction is the effect of magnetic moment gain. This effect occurs for both the on-off and proportional systems. For gains that are too low, transient performance is slow. For high gains, transient performance is good for most orientations of initial wheel angular momentum. However, when the angular momentum vector becomes approximately alined with the earth's magnetic field, a high gain can cause the performance to deteriorate, since the applied torque keeps the wheel angular momentum vector too closely alined with the earth's magnetic field vector. Thus, care must be used in the selection of gain so that performance is adequate for all orientations of wheel angular momentum. The use of a dead zone in the control function will alleviate this problem when as high a gain as possible is desired. Hence, even for a system with equal control system parameters for each control axis, the transient response characteristics will vary for each axis since they will also depend upon the magnetic field variation.

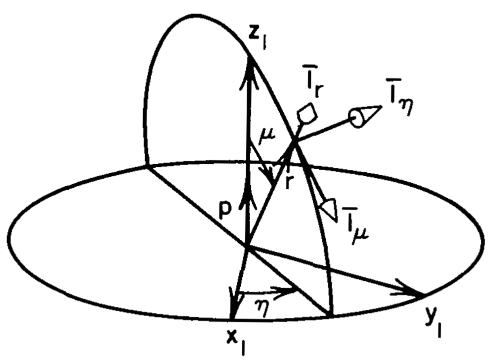
The analysis indicated that the adaptive system would give identical performance with the magnetometer system so long as system imperfections and adaptive system test inputs were neglected. The adaptive system negates the need for a magnetometer. On the other hand, an adaptive system needs greater accuracy for its momentum wheel-speed measurements and needs the components for a rudimentary sampled data system. Hence, the adaptive system is most applicable to vehicles such as the Orbiting Astronomical Observatory in which precise momentum-wheel control and analogous signal processing equipment are already required.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif., Dec. 3, 1964

APPENDIX

AN APPROXIMATION OF THE EARTH'S MAGNETIC FIELD VARIATION FOR AN INERTIALLY ORIENTED VEHICLE

To obtain a representation of the earth's magnetic field as a constant magnitude vector which rotates at a constant angular rate, the following approximations have first been made. The earth's field is represented by a simple magnetic dipole and the magnetic pole is assumed to be at the geographic pole. With these assumptions, the earth's magnetic field is represented by an inertially fixed dipole. The dipole of strength p is alined with the z_1 axis of an inertially fixed x_1, y_1, z_1 coordinate system. The magnetic field, $\bar{B}$, can be expressed in the r, μ, η spherical coordinate system shown in sketch (c). The equations are (e.g., ref. 11)



Sketch (c)

$$\left.\begin{aligned} B_r &= \frac{2p}{r^3}\cos\mu \\ B_\mu &= \frac{p}{r^3}\sin\mu \\ B_\eta &= 0 \end{aligned}\right\} \tag{A1}$$

Components of the magnetic field in the r, μ, η directions for a point given by the r, μ, η coordinates can be transformed to the x_1, y_1, z_1 components by the following transformation:

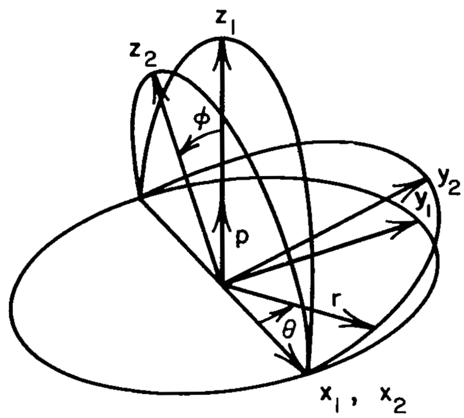
$$\begin{bmatrix} B_{x_1} \\ B_{y_1} \\ B_{z_1} \end{bmatrix} = \begin{bmatrix} \sin\mu\cos\eta & \cos\mu\cos\eta & -\sin\eta \\ \sin\mu\sin\eta & \cos\mu\sin\eta & \cos\eta \\ \cos\mu & -\sin\mu & 0 \end{bmatrix} \begin{bmatrix} B_r \\ B_\mu \\ B_\eta \end{bmatrix} \tag{A2}$$

The substitution of equations (A1) into equation (A2) yields

$$\left.\begin{aligned} B_{x_1} &= \frac{3p}{r^3}\sin\mu\cos\mu\cos\eta \\ B_{y_1} &= \frac{3p}{r^3}\sin\mu\cos\mu\sin\eta \\ B_{z_1} &= \frac{3p}{r^3}\left(-\frac{1}{3} + \cos^2\mu\right) \end{aligned}\right\} \tag{A3}$$

Next the x_1, y_1, z_1 magnetic field components are obtained for a point described by the r, θ, φ coordinates shown in sketch (d).

This system is more convenient for the representation of orbit motion. The axis x_1 is selected to be the line of nodes. The orientation of the x_2, y_2, z_2 axis system is given by a rotation about the x_1 axis through the angle φ. The orbital plane is the $x_2 y_2$ plane. The position of the vehicle in the orbital plane is given by θ, a rotation about the z_2 axis. For a circular orbit (constant r), θ varies at a constant rate, $\dot{\theta}_0$. The following relationships between angles in the r, μ, η coordinate system and the r, θ, φ system can be obtained from the expressions for components of a radial vector in the x_1, y_1, z_1 directions:



Sketch (d)

$$\left.\begin{aligned} \cos\theta &= \sin\mu\cos\eta \\ \sin\theta\cos\varphi &= \sin\mu\sin\eta \\ \sin\theta\sin\varphi &= \cos\mu \end{aligned}\right\} \tag{A4}$$

The substitution of equations (A4) into equations (A3) gives the following relations for components of the magnetic field in the x_1, y_1, z_1 direction for a point described in the r, θ, φ system

$$\left.\begin{aligned} B_{x_1} &= \frac{3p}{r^3}\cos\theta\sin\theta\sin\varphi \\ B_{y_1} &= \frac{3p}{r^3}\sin^2\theta\cos\varphi\sin\varphi \\ B_{z_1} &= \frac{3p}{r^3}\left(-\frac{1}{3}+\sin^2\theta\sin^2\varphi\right) \end{aligned}\right\} \tag{A5}$$

Next, an orthogonal transformation is selected for which the magnetic field along one axis is independent of the angle θ. The transformation from the x_1, y_1, z_1 axis system to the x_2, y_2, z_2 system is used for this purpose.

$$\begin{bmatrix} B_{x_2} \\ B_{y_2} \\ B_{z_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & \sin\varphi \\ 0 & -\sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} B_{x_1} \\ B_{y_1} \\ B_{z_1} \end{bmatrix} \tag{A6}$$

The substitution of equations (A5) into equation (A6) yields

$$\left.\begin{aligned} B_{x_2} &= \frac{3}{2}\frac{p}{r^3}\sin\varphi\sin 2\theta \\ B_{y_2} &= \frac{3}{2}\frac{p}{r^3}\sin\varphi\left[\frac{1}{3} - \cos 2\theta\right] \\ B_{z_2} &= \frac{-p}{r^3}\cos\varphi \end{aligned}\right\} \tag{A7}$$

For a vehicle in a circular orbit with a constant angular rate $\dot{\theta}_o$, a portion of the magnetic field components varies with a frequency of twice this angular rate. If the 1/3 factor in the B_{y_2} equation is neglected, equations (A7) become

$$\left.\begin{aligned} B_{x_2} &= \frac{3}{2}\frac{p}{r^3}\sin\varphi\sin 2\theta \\ B_{y_2} &= -\frac{3}{2}\frac{p}{r^3}\sin\varphi\cos 2\theta \\ B_{z_2} &= -\frac{p}{r^3}\cos\varphi \end{aligned}\right\} \tag{A8}$$

With this approximation and for a fixed inclination angle φ, the B_{z_2} component is constant and the B_{x_2} and B_{y_2} components represent a constant magnitude vector, $(3p/2r^3)\sin\varphi$, which rotates at twice the orbital rate. An x_3, y_3, z_3 rotating coordinate system is selected (for convenience) with x_3 alined with the fixed magnetic field component and z_3 alined with the rotating component. The transformation to the x_3, y_3, z_3 system is as follows:

$$\begin{bmatrix} B_{x_3} \\ B_{y_3} \\ B_{z_3} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ \cos 2\theta & \sin 2\theta & 0 \\ \sin 2\theta & -\cos 2\theta & 0 \end{bmatrix} \begin{bmatrix} B_{x_2} \\ B_{y_2} \\ B_{z_2} \end{bmatrix} \tag{A9}$$

Through use of equations (A8) and (A9), the resulting magnetic field components expressed in the rotating x_3, y_3, z_3 system are found to be

$$\left.\begin{aligned} B_{x_3} &= \frac{p}{r^3}\cos\varphi \\ B_{y_3} &= 0 \\ B_{z_3} &= \frac{3}{2}\frac{p}{r^3}\sin\varphi \end{aligned}\right\} \qquad \text{(A10)}$$

The rotating component of the magnetic field, B_{z_3}, corresponds to the B_r term used in the body of the report.

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TABLE I.- VALUES OF PARAMETERS USED IN THE CALCULATED RESULTS

Magnetic Field

$\bar{B}_1$, gauss

$B_x = 0.2 + 0.12 \sin(\gamma t + 90^\circ)$

$B_y = 0.09 + 0.16 \sin(\gamma t - 90^\circ)$

$B_z = 0.2 \sin(\gamma t + 180^\circ)$

$\bar{B}_2$, gauss

$B_x = 0.215$

$B_y = 0.20 \sin(\gamma t - 90^\circ)$

$B_z = 0.20 \sin(\gamma t + 180^\circ)$

$\bar{B}_3$, gauss

$B_x = 0.215 + 0.043 \sin \gamma t$

$B_y = 0.20 \sin(\gamma t - 90^\circ)$

$B_z = -0.04 + 0.20 \sin(\gamma t + 180^\circ)$

External Disturbance

$\bar{t}_{e_1} \left(\frac{1}{\text{orbital period}}\right)$

$t_{ex} = 0.2 + 0.3 \sin(\gamma t + 30^\circ)$

$t_{ey} = 0.8 + 0.1 \sin \gamma t$

$t_{ez} = 0.8 + 0.4 \sin(\gamma t + 60^\circ)$

$\bar{t}_{e_2} \left(\frac{1}{\text{orbital period}}\right)$

$t_{ex} = 0.73$

$t_{ey} = 0.68 \sin(\gamma t - 90^\circ)$

$t_{ez} = 0.68 \sin(\gamma t + 180^\circ)$

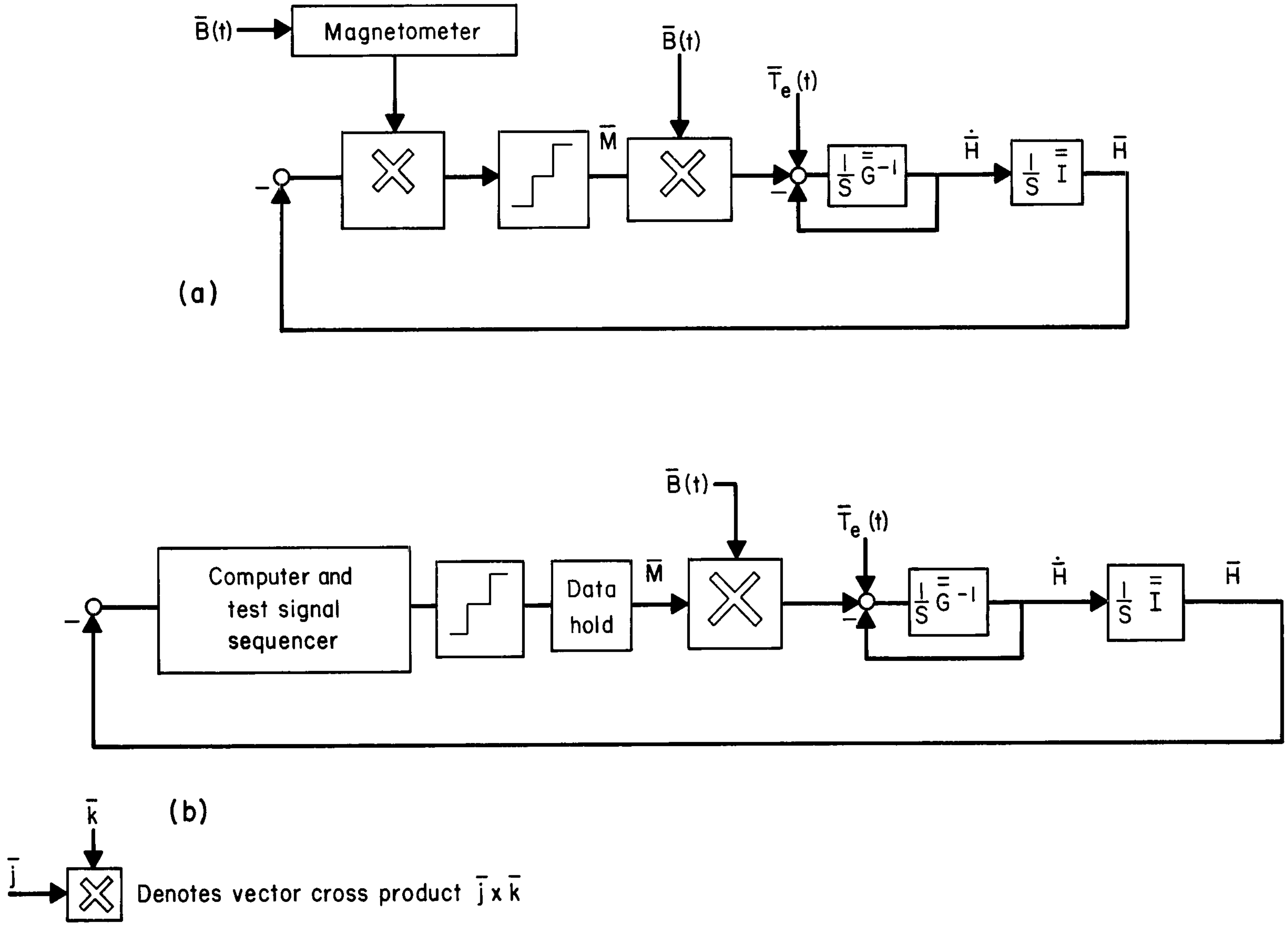
Initial Conditions

$\bar{h}(0)_1$

$h_x = 0.20$

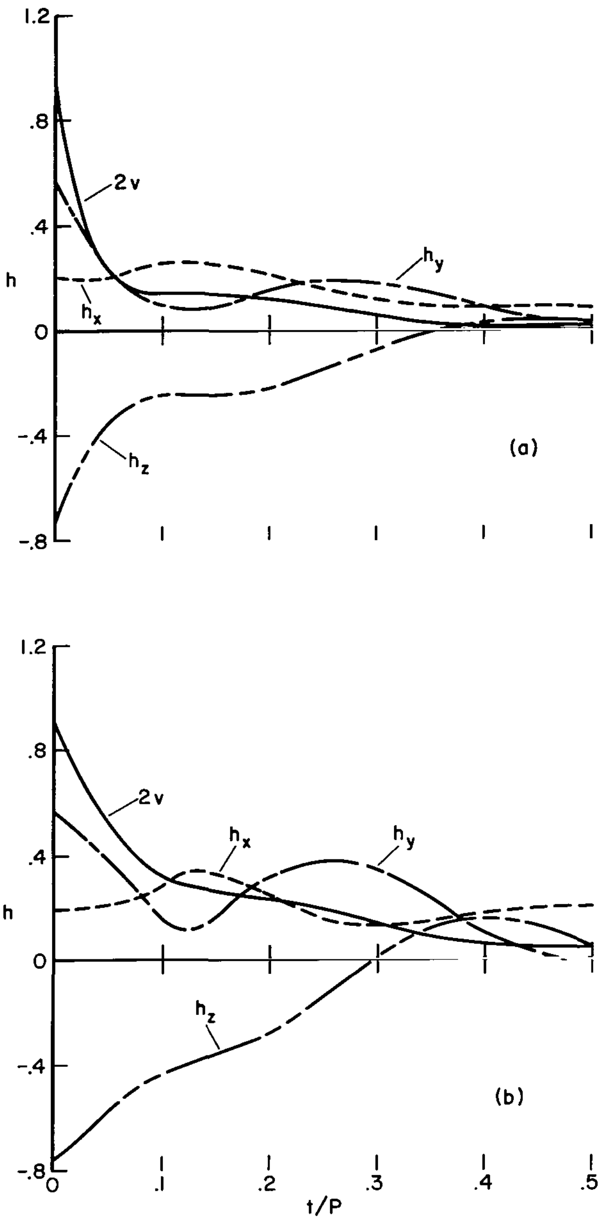
$h_y = 0.55$

$h_z = -0.75$



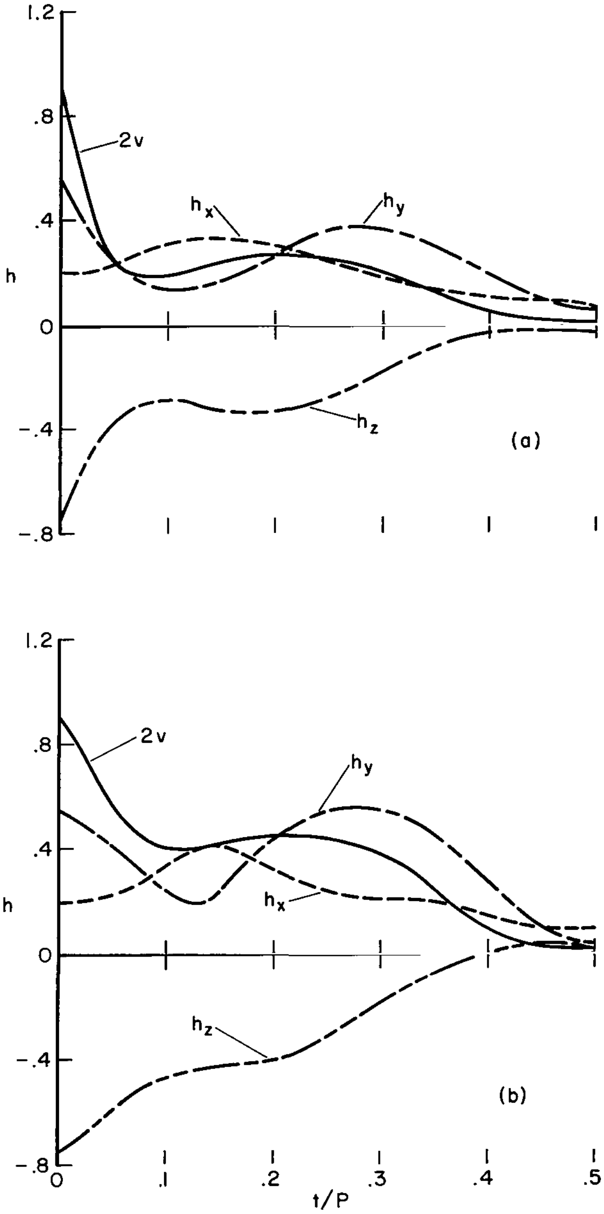
(a) Magnetometer on-off system.
(b) Adaptive on-off system.

Figure 1.- Block diagrams of magnetic torquer systems.



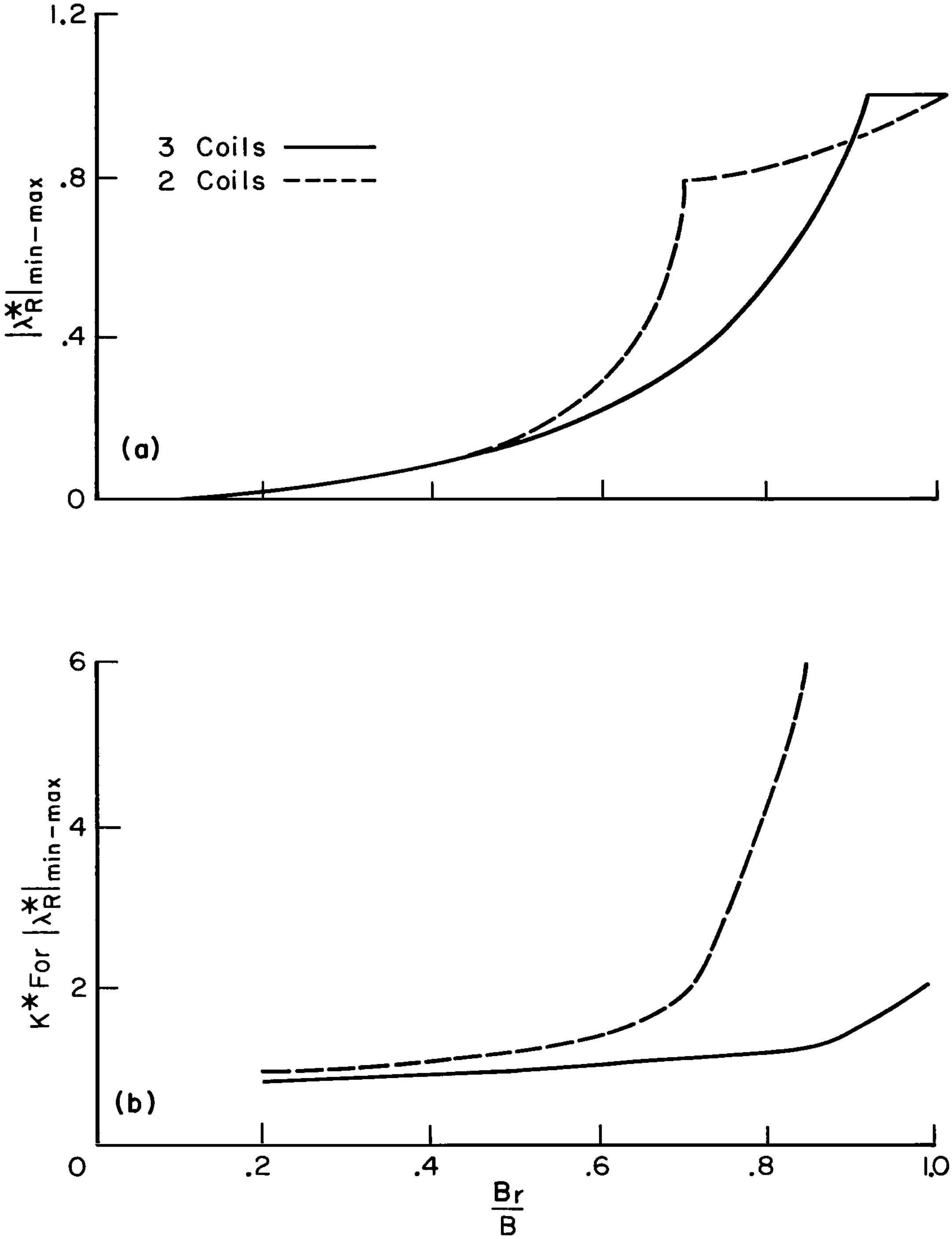
(a) Proportional system, k = 156.
(b) On-off system, n = 12.5.

Figure 2.- Comparison of transient response for proportional and on-off systems, $\overline{B}_1$.



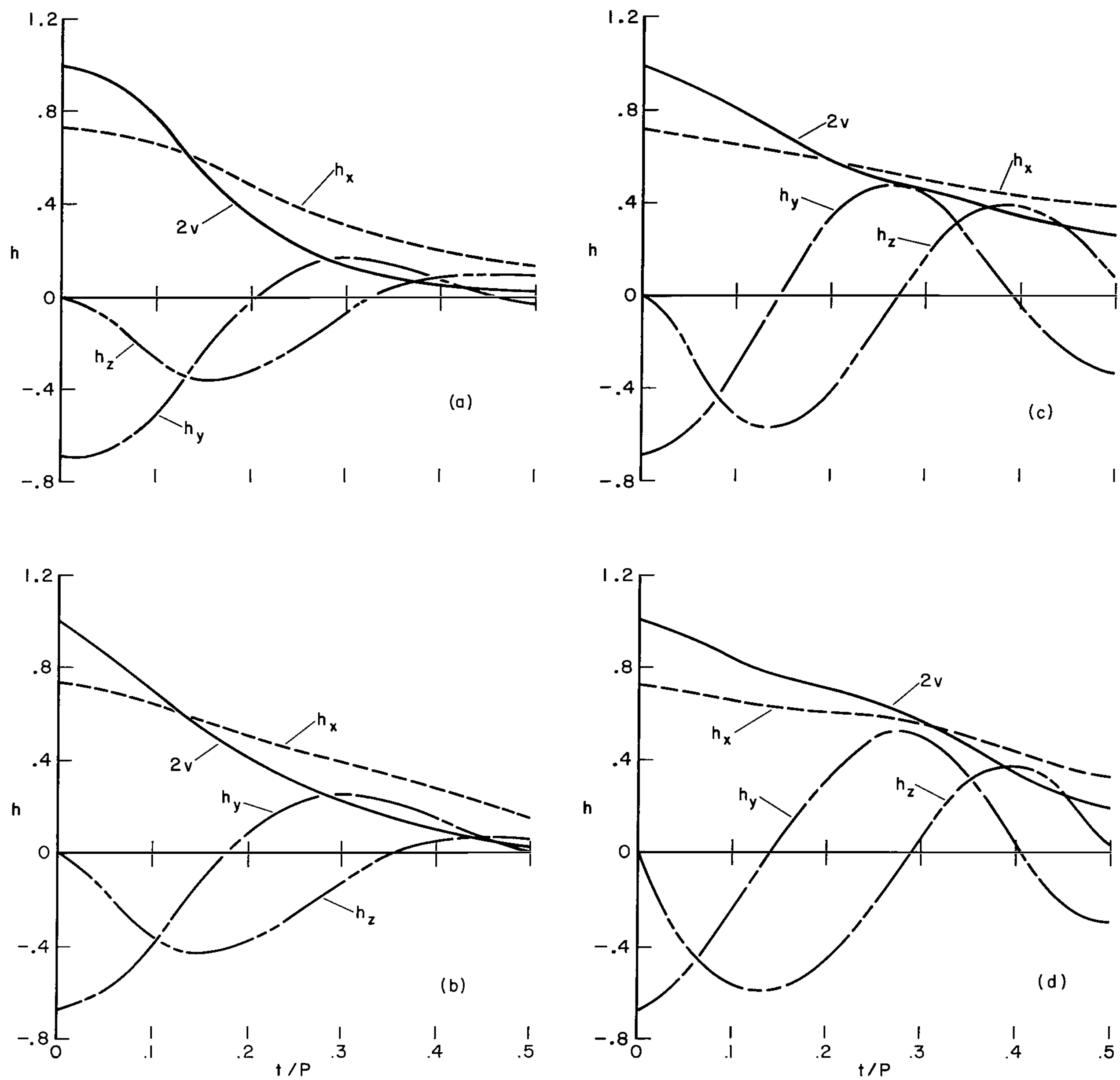
(a) Proportional system, k = 156.
(b) On-off system, n = 12.5.

Figure 3.- Comparison of transient response for proportional and on-off systems with external disturbance, $\overline{B}_1, \overline{t}_{e_1}$.



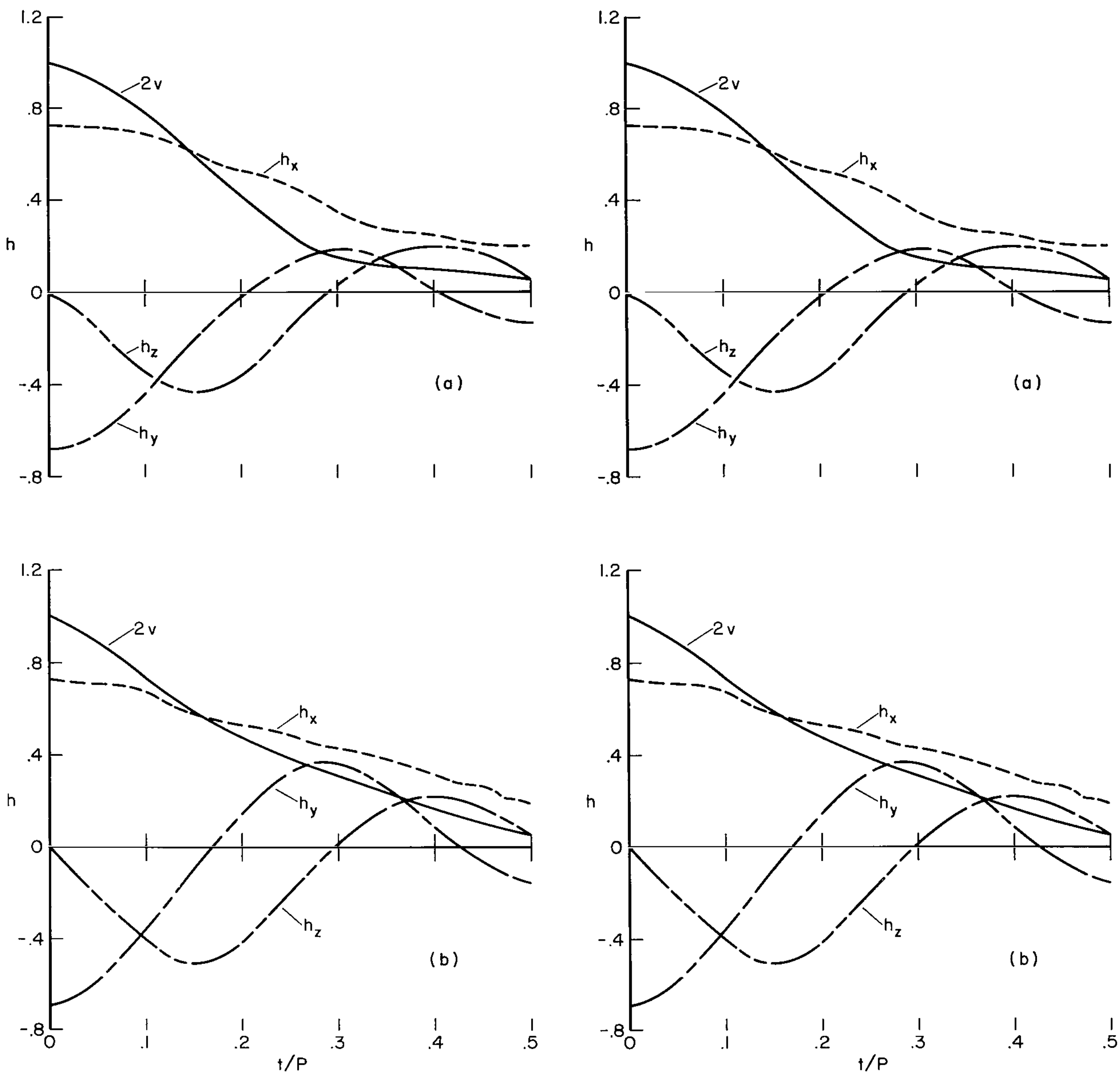
(a) Maximum value of minimum root.
(b) Gain required for maximum-minimum root.

Figure 4.- Effect of ratio of fixed to rotating components of the earth's magnetic field on the system as obtained from the simplified analysis.



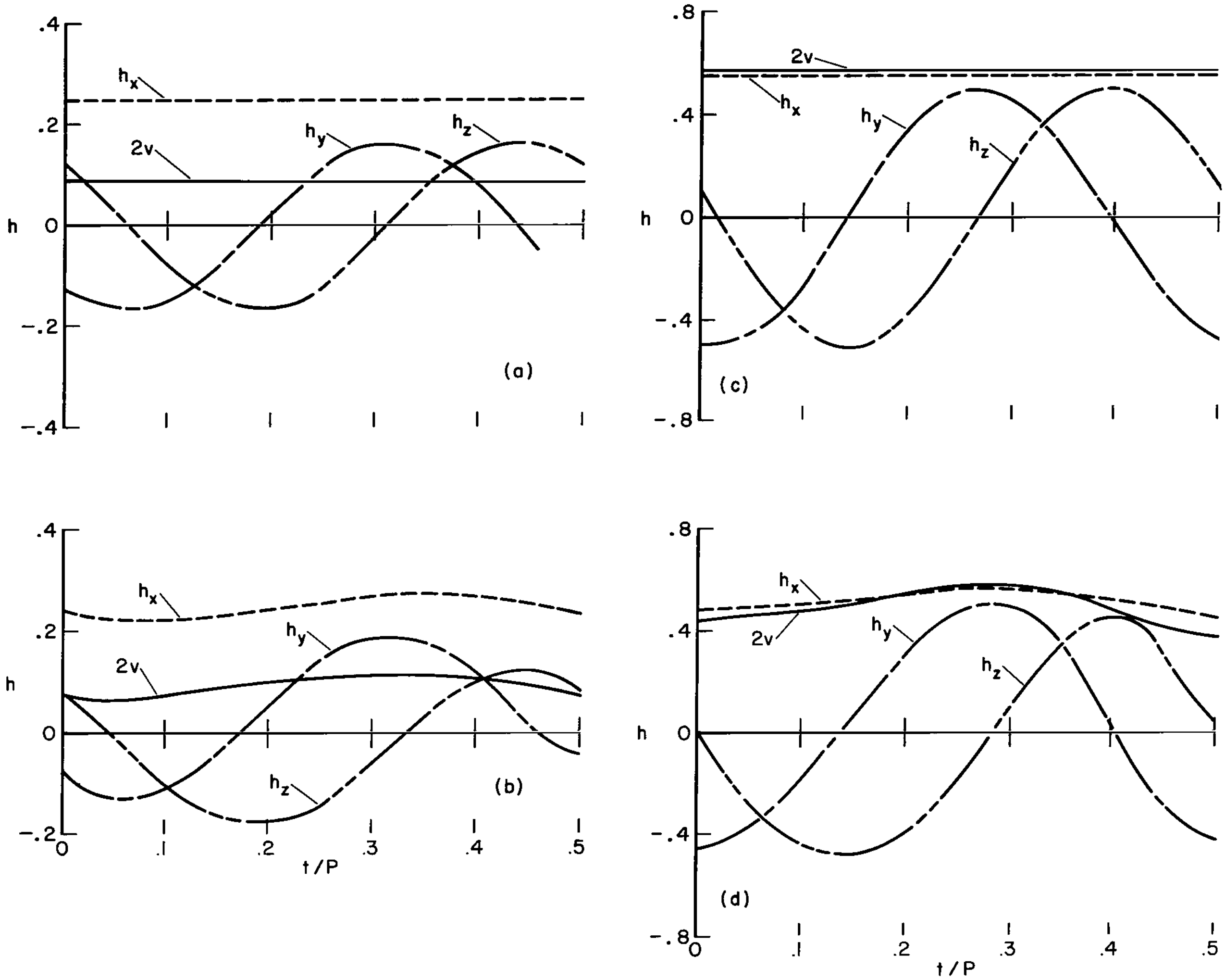
(a) Moderate gain, $|\overline{B}|$ constant, $\overline{B}_2$, $k = 156$.
(b) Moderate gain, $|\overline{B}|$ varies, $\overline{B}_3$, $k = 156$.
(c) High gain, $|\overline{B}|$ constant, $\overline{B}_2$, $k = 624$.
(d) High gain, $|\overline{B}|$ varies, $\overline{B}_3$, $k = 624$.

Figure 5.- Effect of gain on transient response of proportional system.



(a) Moderate gain, $|\overline{B}|$ constant, $\overline{B}_2$, n = 12.5.
(b) Moderate gain, $|\overline{B}|$ varies, $\overline{B}_3$, n = 12.5.
(c) High gain, $|\overline{B}|$ constant, $\overline{B}_2$, n = 50.
(d) High gain, $|\overline{B}|$ varies, $\overline{B}_3$, n = 50.

Figure 6.- Effect of gain on transient response of on-off system.



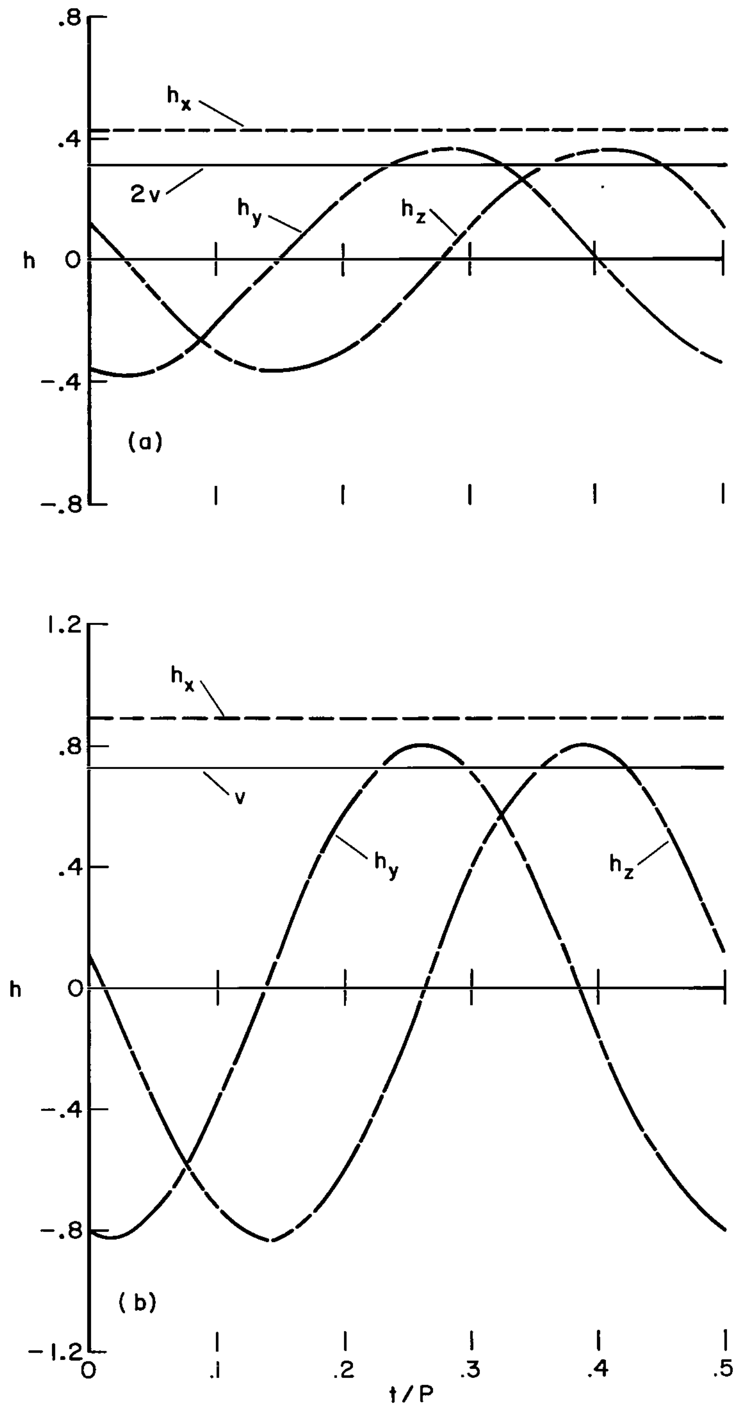
(a) Moderate gain, $|\overline{B}|$ constant, $\overline{B}_2$, k = 156.

(b) Moderate gain, $|\overline{B}|$ varies, $\overline{B}_3$, k = 156.

(c) High gain, $|\overline{B}|$ constant, $\overline{B}_2$, k = 624.

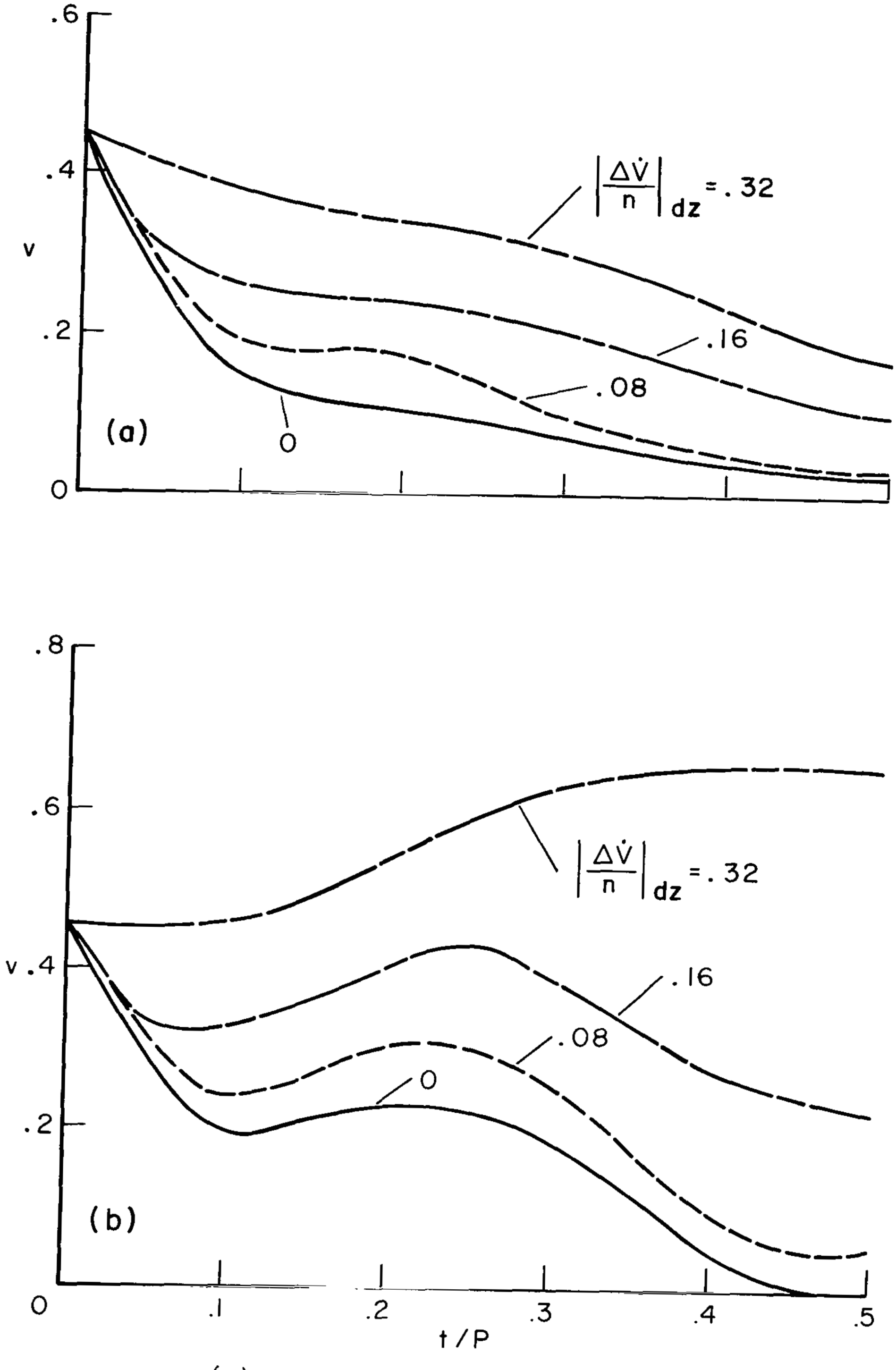
(d) High gain, $|\overline{B}|$ varies, $\overline{B}_3$, k = 624.

Figure 7.- Effect of gain on steady-state response with external disturbance for proportional system, $\overline{t}_{e_2}$.



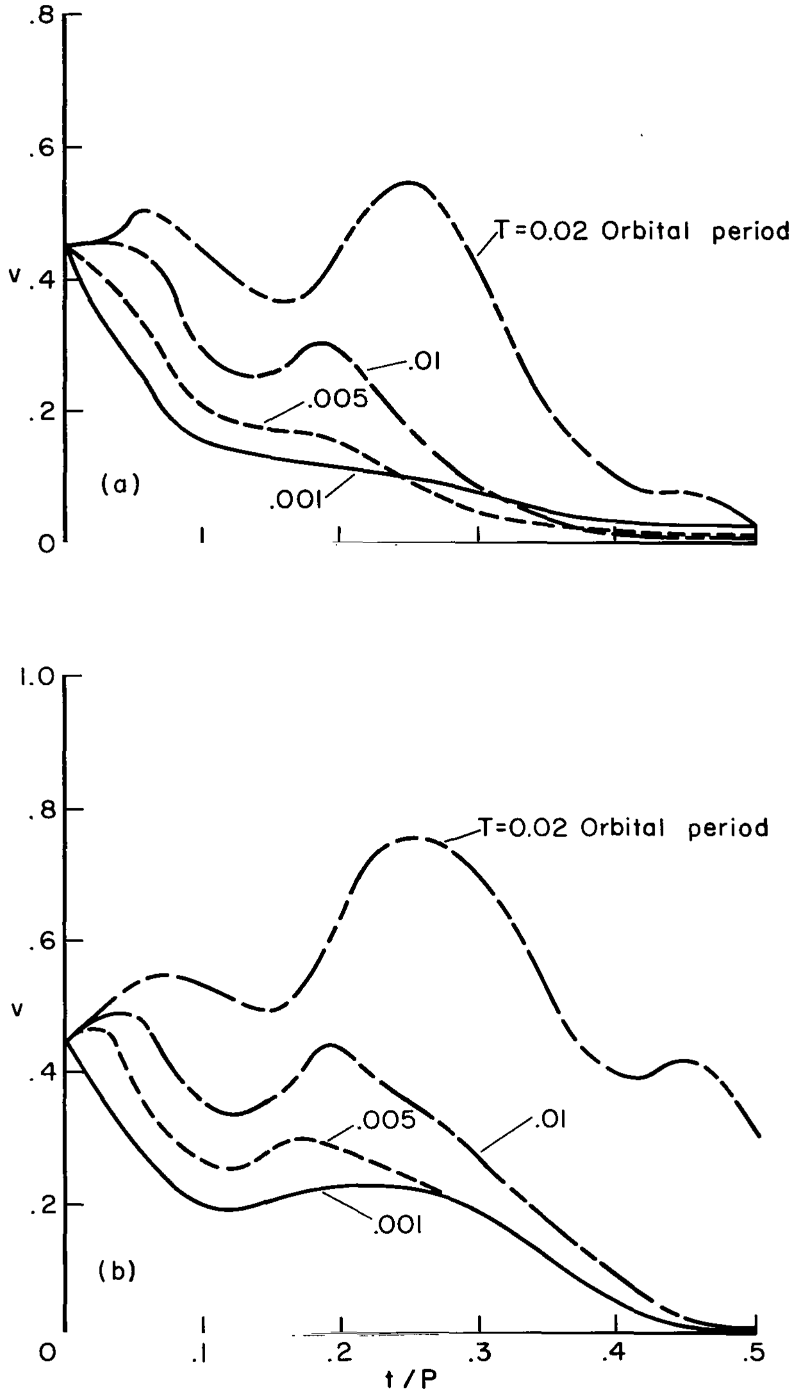
(a) Moderate gain, n = 12.5.
(b) High gain, n = 50.

Figure 8.- Effect of gain on steady-state response with external disturbance for on-off system, $\overline{B}_2$, $\overline{t}_{e_2}$.



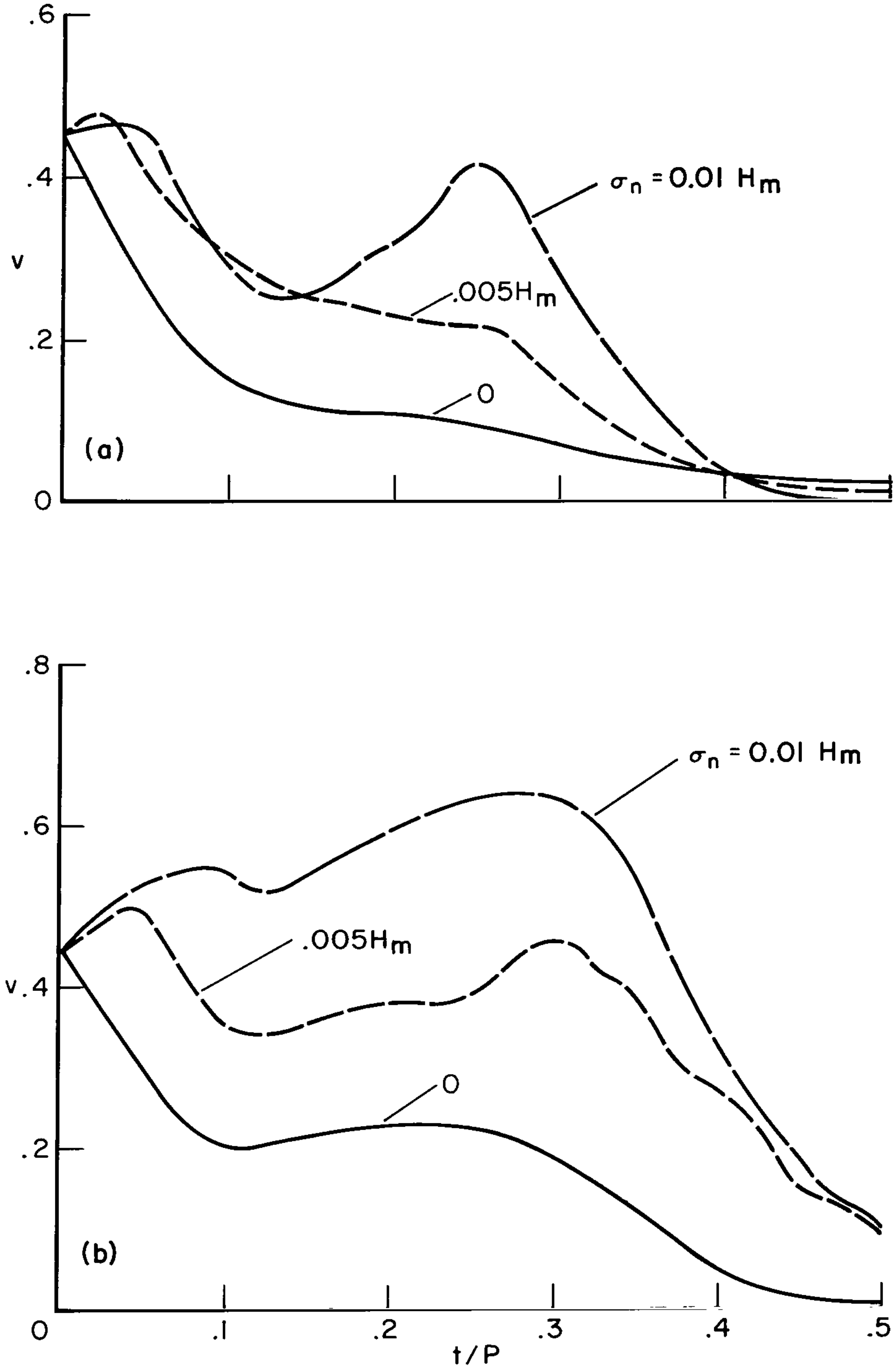
(a) No external disturbance.
(b) With external disturbance, $\bar{t}_{e_1}$.

Figure 9.- Effect of dead zone on adaptive on-off system performance, $\bar{B}_1$, $\bar{h}(0)_1$, $n = 12.5$.



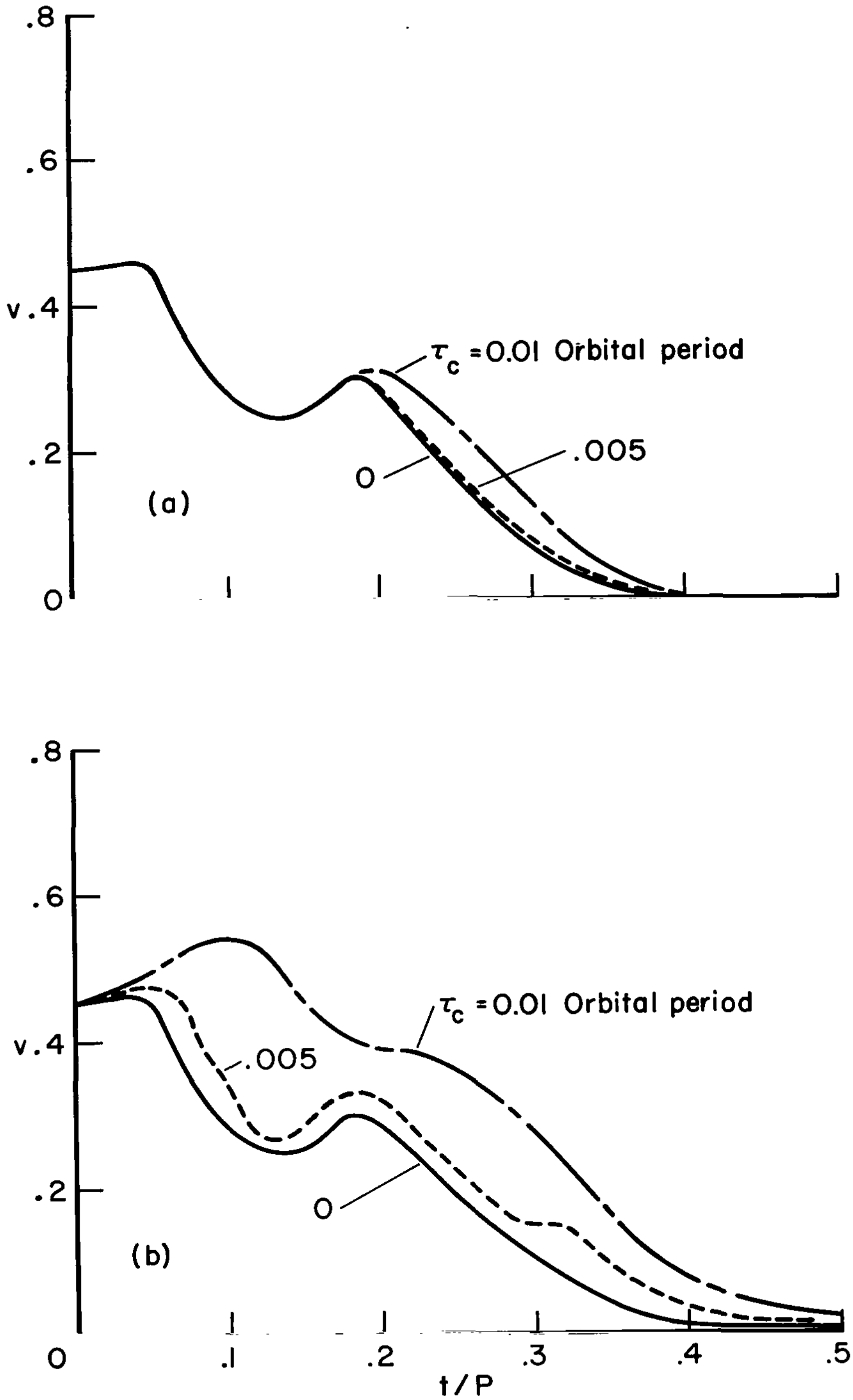
(a) No external disturbance.
(b) With external disturbance, $\overline{t}_{e_1}$.

Figure 10.- Effect of sampling interval on on-off adaptive system performance, $\overline{B}_1$, $\overline{h}(0)_1$, n = 12.5.



(a) No external disturbance.
(b) With external disturbance, $\bar{t}_{e_1}$.

Figure 11.- Effect of wheel-speed measurement error on on-off adaptive system performance, $\bar{B}_1$, $\bar{h}(0)_1$, n = 12.5.



(a) No measurement error.
(b) 1/2-percent rms measurement error, $\sigma_n = 0.005\ H_m$.

Figure 12.- Effect of momentum wheel control system time constant on on-off adaptive system, $T = 0.01$, $\overline{B}_1$, $\overline{h}(0)_1$, $n = 12.5$.

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